Pro Tip: Volumes & Areas are never negative.

Math 2D Afternoon Last Quiz - March 10th

Please put ID on back for redistribution!

Show all of your work. *There is a question on the back side.

1. You will be finding the volume of the following:

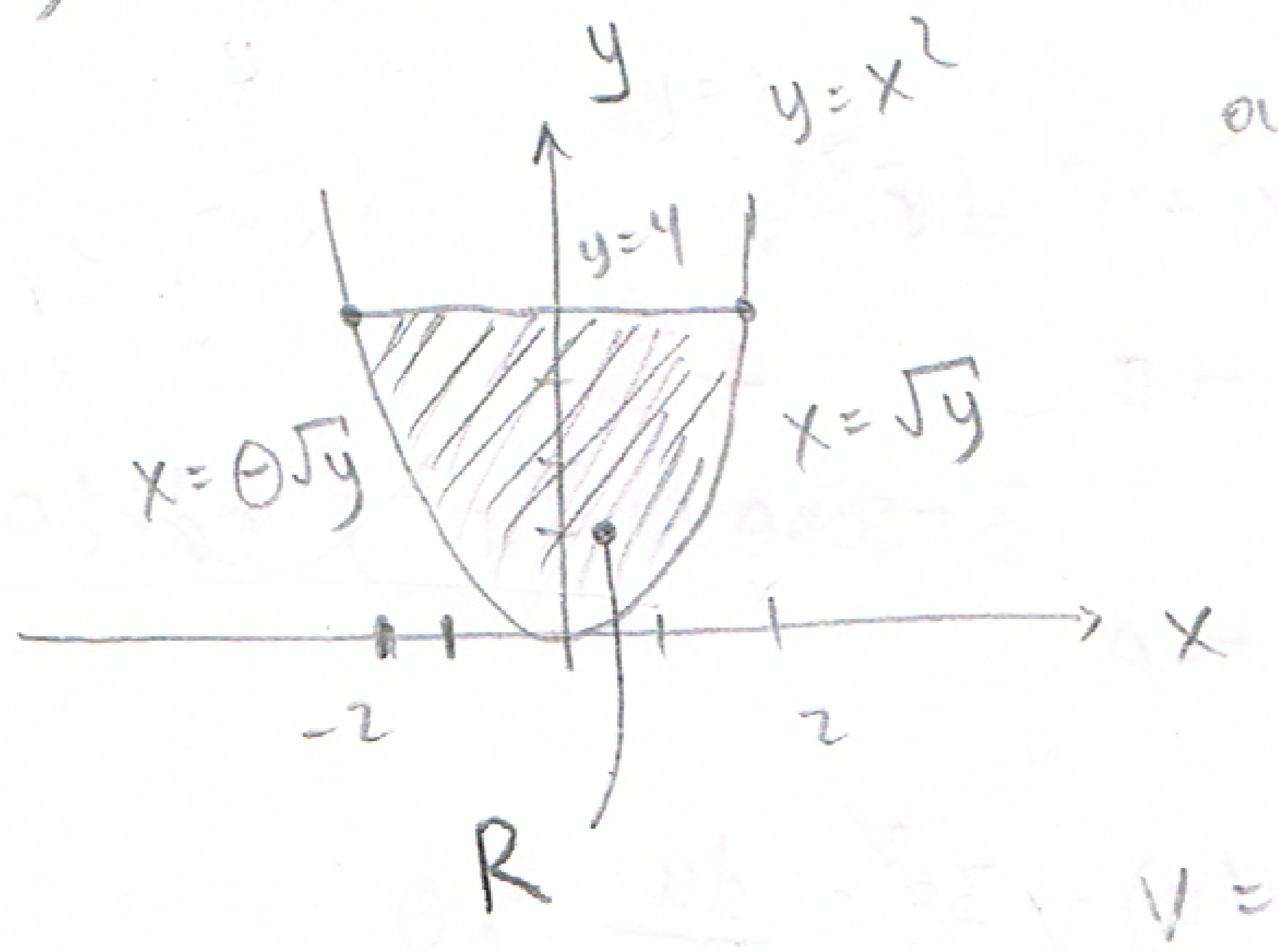
The volume enclosed by the cylinders $z = x^2 - 2, y = x^2$, and the planes $z = -2, y = 4$.

(a) Write the integral that gives your volume in both ways, as $\iint_R f(x,y)dxdy$ and $\iint_R f(x,y)dydx$.

(b) Use one of the ways above to compute the volume. You must simplify your answer to a single fraction. (For this reason, one of the ways above is easier for simplifying).

Hint: Is there any nice symmetry?

a) We see:



When $y=x^2 \Rightarrow x=\pm\sqrt{y}$. And second, our difference in height is $(x^2-2)-(-2)$ from z's. (Like $z_{top}-z_{bot}$)

So, $V = \iint_R (x^2 \cancel{-2} \cancel{+2})\, dA$.

We are to write it in 2 ways,

$$V = \int_{x=-2}^{x=2}\int_{y=x^2}^{y=4} x^2\, dy\, dx = \int_{y=0}^{y=4}\int_{x=-\sqrt{y}}^{x=\sqrt{y}} x^2\, dx\, dy.$$

+1 +1

b)

$$V = \int_{-2}^{2}\int_{x^2}^{4} x^2\, dy\, dx$$

$$= \int_{-2}^{2} x^2 y \Big|_{y=x^2}^{y=4} dx \quad +1$$

$$= \int_{-2}^{2} (4x^2 - x^4)\, dx$$

Even fn, symmetry

$$\overset{\text{}}{=} 2\int_0^2 (4x^2 - x^4)\, dx \quad +1$$

$$= 2\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]\Big|_{x=0}^{x=2}$$

$$= 2\left[\frac{2^2 \cdot 2^3}{3} - \frac{2^5}{5}\right]$$

$$= 2\left[\frac{5\cdot 2^5 - 3\cdot 2^5}{15}\right] = 2\left[\frac{2\cdot 2^5}{15}\right]$$

OR

$$V = \int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} x^2\, dx\, dy \quad +1$$

symmetry

$$= \int_0^4 2\left(\frac{x^3}{3}\right)\Big|_{x=0}^{x=\sqrt{y}} dy$$

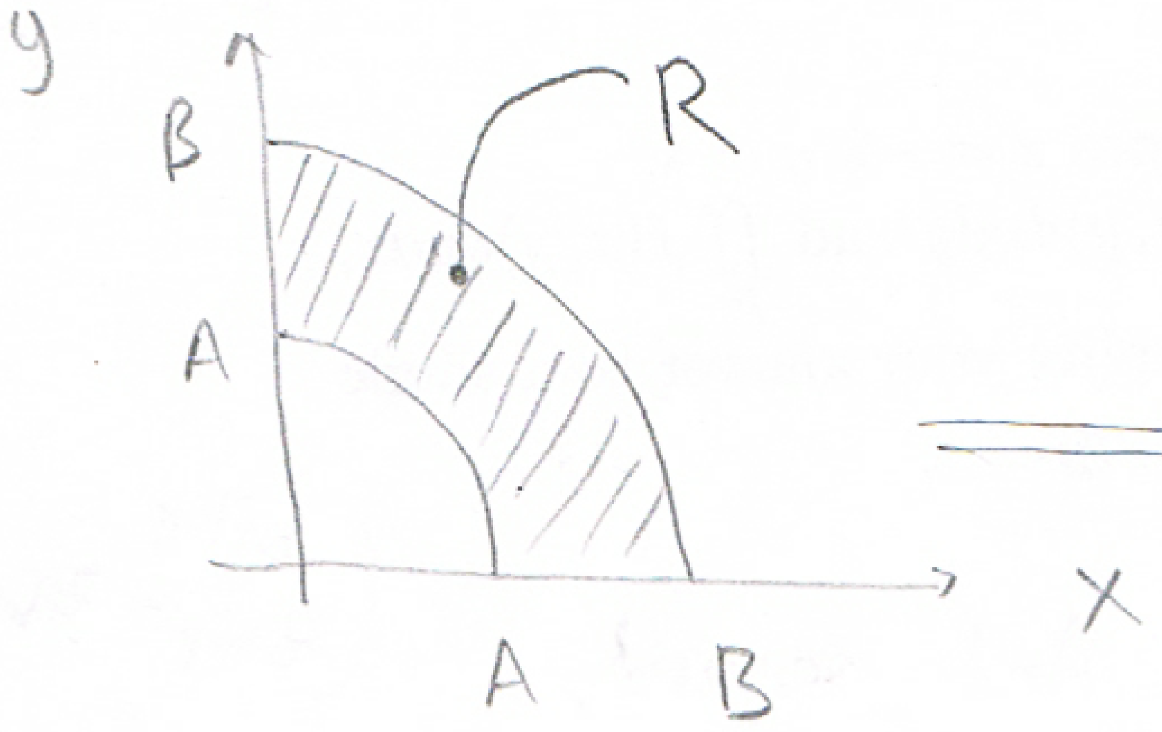
$$= \int_0^4 \frac{2y^{3/2}}{3}\, dy \quad +1$$

$$= \frac{2}{3}\cdot y^{5/2}\cdot \frac{2}{5}\Big|_{y=0}^{y=4}$$

$$= \frac{4}{15}\cdot 4^{5/2} = \frac{4\cdot 2^5}{15} = \frac{2^2\cdot 2^5}{15}$$

$$= \boxed{\frac{2^7}{15} = \frac{128}{15}} \quad +1$$

2. Compute $\iint\limits_R 3x\sin((x^2+y^2)^{3/2})dA$ where R is the region in the first quadrant $(x>0, y>0)$ between the circles $x^2+y^2=A^2$ and $x^2+y^2=B^2$ where $0<A<B$.



- We have a circular domain (and we have "x^2+y^2" - terms in sine).

$\longrightarrow$ ↳ Use polar, so $\boxed{\begin{array}{l} 0 \le \theta \le \pi/2 \\ A \le r \le B \end{array}}$

- our $x = r\cos\theta$, $y = r\sin\theta$, so $x^2+y^2=r^2$,

+2

$$\iint_R 3x\sin\left((x^2+y^2)^{3/2}\right) dA = \int_{\theta=0}^{\theta=\frac{\pi}{2}}\int_{r=A}^{r=B} 3r\cos\theta \sin(r^3)\, r\,dr\,d\theta$$

$u=r^3$, $du=3r^2dr$ (so $dr=\frac{du}{3r^2}$)

* Also, bounds become $u(r=A)=A^3$, $u(r=B)=B^3$

$$\overset{u\text{-sub}}{=} \int_0^{\pi/2}\int_{A^3}^{B^3} \cos\theta \sin(u)\cdot \cancel{3r^2}\cdot \frac{du}{\cancel{3r^2}}\, d\theta$$

+2

$$= \int_0^{\frac{\pi}{2}} \cos\theta \left(-\cos u\right)\Big|_{u=A^3}^{u=B^3} d\theta$$

$$= \int_0^{\pi/2} \cos\theta \left(\underbrace{\cos(A^3)-\cos(B^3)}_{\text{constant!}}\right) d\theta$$

$$= \left[\cos(A^3)-\cos(B^3)\right]\cdot\left(\sin\theta\right)\Big|_{\theta=0}^{\theta=\pi/2}$$

+1

$$= \boxed{\cos(A^3)-\cos(B^3)}$$